Properties of vector Addition and Scalar Multiplication

Let **u**, **v**, and **w** be vectors and let c and d be scalars. Then the following properties are true.

1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$	2. $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$	3. $\mathbf{u}+0=\mathbf{u}$
4. $\mathbf{u}+(-\mathbf{u})=0$	5. $c(d\mathbf{u})=(cd)\mathbf{u}$	6. $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$
7. $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$	8. $1(\mathbf{u})=\mathbf{u}$, $0(\mathbf{u})=0$	9. $\|c v\|=|c|\|v\|$

Unit Vectors

In many applications, it is useful to find a unit vector that has the same direction as a given nonzero vector **v**. To do this, you can divide **v** by its magnitude to obtain

$$u = unit\ vector = \frac{v}{\|v\|} = \left(\frac{1}{\|v\|}\right) v$$

Note that **u** is a scalar multiple of **v**. The vector **u** has a magnitude of **1** and the same direction as **v**. The vector **u** is called a unit vector in the direction of v.

Ex: 4 Find a unit vector in the direction of $\mathbf{v}=\langle 7,-3\rangle$ and verify that the result has a magnitude 1.

$\|v\| = \sqrt{(7)^2+(-3)^2}$

$= \sqrt{49+9}$

$\|v\| = \sqrt{58}$

$u = \frac{V}{\|V\|} = \frac{1}{\|V\|}\cdot V$

$u = \left\langle \frac{7}{\sqrt{58}}, \frac{-3}{\sqrt{58}} \right\rangle$

$\|u\| = \sqrt{\left(\frac{7}{\sqrt{58}}\right)^2 + \left(\frac{-3}{\sqrt{58}}\right)^2}$

$\sqrt{\frac{49}{58}+\frac{9}{58}}$

$\sqrt{\frac{58}{58}} = 1$ ✓

You Try: Find a unit vector in the direction of $\mathbf{v}=\langle 6,-1\rangle$ and verify that the result has a magnitude 1.

$\|v\| =$

$u =$

$\|u\| =$

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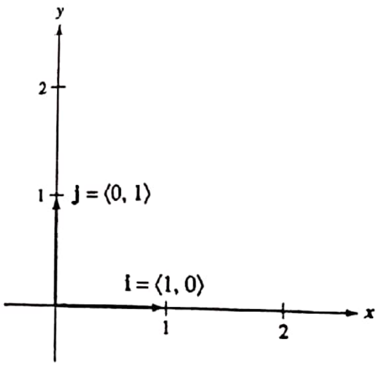


Figure 6.19

The unit vectors $\langle 1, 0 \rangle$ and $\langle 0, 1 \rangle$ are called Standard unit vectors and are denoted by **i** = $\langle 1, 0 \rangle$ and **j** = $\langle 0, 1 \rangle$ as shown in Figure 6.19.

These vectors can be used to represent any vector $\vec{v} = \langle v_1, v_2 \rangle$.

$$\vec{v} = \langle v_1, v_2 \rangle = v_1\langle 1,0 \rangle + v_2\langle 0,1 \rangle = v_1\vec{\imath} + v_2\vec{\jmath}$$

The scalars v_1 *and* v_2 are called the **horizontal** and **vertical** components of $\vec{v}$.

$v_1\vec{\imath} + v_2\vec{\jmath}$ is called a **linear combination** of the vectors $\vec{\imath}$ and $\vec{\jmath}$.

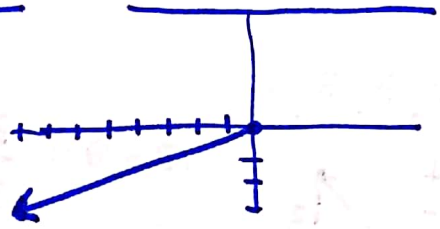
Any vector in the plane can be written as a linear combination of the standard unit vectors $\vec{\imath}$ and $\vec{\jmath}$.

Ex: 5 Let **u** be a vector with initial point P (5, 2) and terminal point Q (-3, -1). Write **u** as a linear combination of the standard unit vectors **i** and **j**.

Component form: $\langle -3-5, -1-2 \rangle$

$\langle -8, -3 \rangle$

$\boxed{-8i - 3j}$



You Try: Let **u** be a vector with initial point (-2, 6) and terminal point (-8, 3). Write **u** as a linear combination of the standard unit vectors **i** and **j**.

$u = \langle -3, 8 \rangle$ $v = \langle 2, -1 \rangle$

Ex: 6 Let **u** = -3**i** + 8**j** and **v** = 2**i** – **j**. Find 2**u** – 3**v**.

$2\langle -3, 8 \rangle - 3\langle 2, -1 \rangle$

$\langle -6, 16 \rangle - \langle 6, -3 \rangle$

$\langle -12, 19 \rangle$

$\boxed{-12i + 19j}$

You Try: Let **u** = **i** – 2**j** and **v** = -3**i** + 2**j**. Find 5**u** – 2**v**.

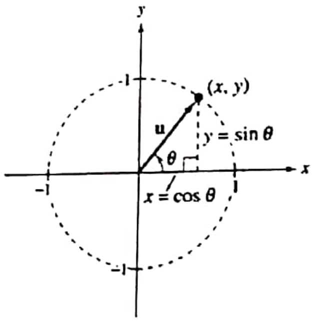
$u = \langle 1, -2 \rangle$ $v = \langle -3, 2 \rangle$

$5\langle 1, -2 \rangle - 2\langle -3, 2 \rangle$

Direction Angles

If **u** is a unit vector such that θ is the angle (measured counterclockwise) from the positive x-axis to **u**, then the terminal point of **u** lies on the unit circle and you have

$$\mathbf{u} = \langle x, y\rangle = \langle \cos\theta, \sin\theta\rangle = (\cos\theta)\boldsymbol{i} + (\sin\theta)\boldsymbol{j}$$ see Figure 6.20



$\|\mathbf{u}\| = 1$

Figure 6.20

The angle θ is the direction angle of the vector **u**.

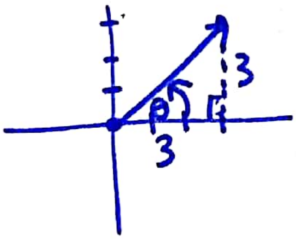
Suppose that **u** is a unit vector with direction angleθ. If **v** = **a**i + **b**j is any vector that makes an angle $\theta$with the positive x-axis, then it has the same direction as **u** and **v** = $\|v\|\langle\cos\theta, \sin\theta\rangle = \|v\|(\cos\theta)\boldsymbol{i} + \|v\|(\sin\theta)\boldsymbol{j}$.

Since **v** = ai + bj = $\|v\|(\cos\theta)\boldsymbol{i} + \|v\|(\sin\theta)\boldsymbol{j}$, then the direction angle θ for **v** is determined from

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\|v\|\sin\theta}{\|v\|\cos\theta} = \frac{b}{a}$$

Ex: 7 Find the direction angle of each vector. a. $\boldsymbol{u} = 3\boldsymbol{i} + 3\boldsymbol{j}$ b. $\boldsymbol{v} = 3\boldsymbol{i} - 4\boldsymbol{j}$

a) $\langle 3, 3\rangle$

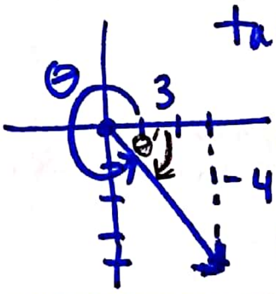


$\tan\theta = \frac{3}{3}$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = 45°}$

b) $\langle 3, -4\rangle$



$\tan\theta' = \frac{-4}{3}$

$\theta' = \tan^{-1}\left(\frac{-4}{3}\right)$

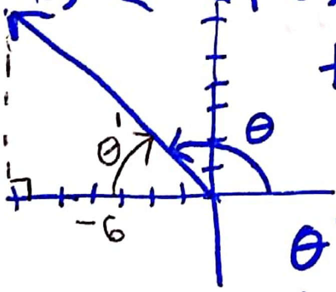
$\theta' = -53.13°$

$+360$

$\boxed{\theta = 306.87°}$

You Try: Find the direction angle of each vector. a. $\boldsymbol{v} = -6\boldsymbol{i} + 6\boldsymbol{j}$ b. $\boldsymbol{v} = -7\boldsymbol{i} - 4\boldsymbol{j}$

a) $\langle -6, 6\rangle$



$\tan\theta' = \frac{6}{-6}$

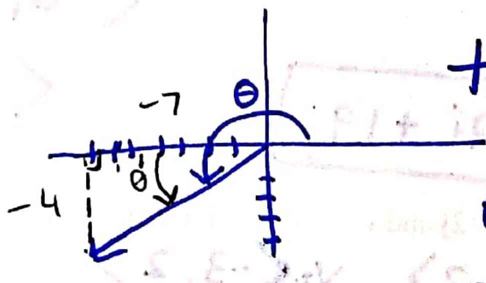
$\theta' = \tan^{-1}(-1)$

$\theta' = -45°$

$\theta = 180 - 45$

$\boxed{\theta = 135°}$

b) $\langle -7, -4\rangle$



$\tan\theta' = \frac{-4}{-7}$

$\theta' = \tan^{-1}\left(\frac{4}{7}\right)$

$\theta' = 29.74°$

$+180$

$\boxed{\theta = 209.74°}$

Horiz. comp. $\|v\|\cos\theta$ vert comp $\|v\|\sin\theta$

The **velocity** of a moving object is a vector because velocity has both magnitude and direction. The magnitude of velocity is **speed**.

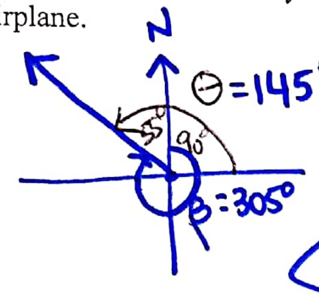
Bearing is the angle that the line of travel makes with due North, measured clockwise. Bearing and the direction angle are not the same angle!

β

θ = direction angle.

Ex. 8 A DC-10 jet aircraft is flying on a bearing of 305° at 520 mph. Find the component form of the velocity of the airplane.

magnitude



$\|v\| = 520$ mph $\quad \theta = 145°$

$\langle \|v\|\cos\theta,\ \|v\|\sin\theta \rangle$

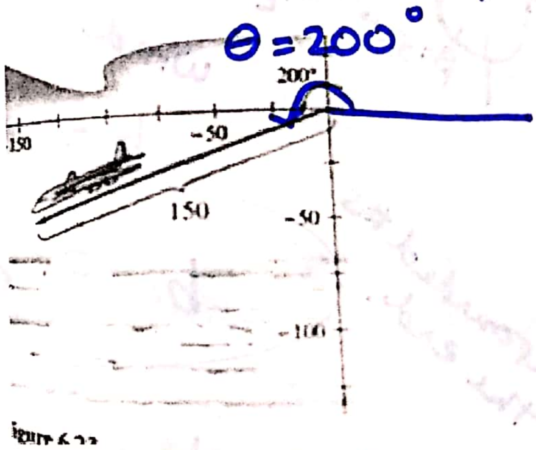
$\langle 520\cos 145°,\ 520\sin 145° \rangle$

$\boxed{\langle -425.96,\ 298.26 \rangle} = -425.96i + 298.26j$

Ex: 9 Find the component form of the vector that represents the velocity of an airplane descending at a speed of 150 miles per hour at an angle 20° below the horizontal.

$\|v\| = 150$ mph

$\theta = 200°$



$\langle 150\cos 200,\ 150\sin 200 \rangle$

$\langle -140.95,\ -51.30 \rangle$

$-140.95i - 51.30j$

Ex: 10 An airplane is traveling at a speed of 500 miles per hour with a bearing of 330°. During take-off, the plane encounters a 70 mph wind in the direction N 45° E. Find the ground speed and direction of the airplane.

Bearing!! :)

1st) Draw a diagram.

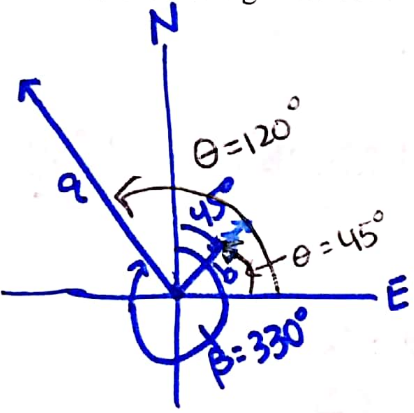
2nd) Find the component form velocity of the airplane without the effect of wind. (a)

3rd) Find the component form velocity of the wind alone (b)

4th) The true velocity of the airplane (with the effect of wind), in component form is v = a + b

5th) The ground speed is the magnitude of v.

6th) The direction of the airplane should be given as a bearing. So, find the direction angle, then convert the direction angle to a bearing.



plane

$a = \langle 500\cos 120, 500\sin 120 \rangle$

$a = \langle -250, 433.01\ldots \rangle$

wind

$b = \langle 70\cos 45°, 70\sin 45° \rangle$

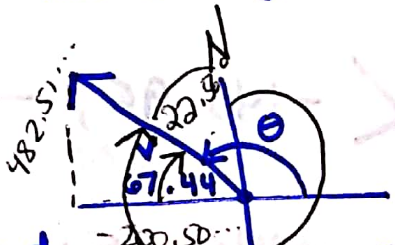
$b = \langle 49.497\ldots, 49.497\ldots \rangle$

use unrounded #s until the end.

$\underline{V} = a + b$ $\quad v = \langle -200.50\ldots, 482.51\ldots \rangle$

$\|v\| = \sqrt{(-200.50\ldots)^2 + (482.51\ldots)^2}$

$= \boxed{522.51 \text{ mph}}$ ← ground speed



$\theta' = \tan^{-1}\left(\frac{482.51\ldots}{-200.50\ldots}\right)$

$\theta' = -67.44$
$+180$
$\theta = 112.56°$
$-90°$

$\beta = 360 - 22.56 =$

or $270 + 67.44 = \boxed{N\ 337.44°E}$

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